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EFFECT OF PERIPHERAL WALL CONDUCTION ON HEAT TRANSFER FROM A CYLINDER IN CROSS FLOW

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Abstract—With uniform heat generation within the wall of the cylinder placed in a cross flow, heat flows by conduction in the peripheral direction due to the asymmetric nature of the fluid flow around the perimeter of the cylinder. The peripheral heat flow affects the wall temperature distribution to such an extent that in some cases significantly different results may be obtained for geometrically similar surfaces.

In the present study, a non-dimensional parameter $K^* = K_\infty R/K_t b$ has been used to characterize the peripheral wall heat conduction. In the experimental investigation five test cylinders having different values of K^* (from 0.00164 to 0.0289) over Reynolds numbers based on the tube diameter varying between about 1000 and 100000 were studied at a fixed turbulence intensity of 4.0% for the free stream.

The large values of K^*, especially at high Reynolds number are found to be associated with large variation of wall temperature. The added asymmetry of the thermal boundary condition affects the average heat-transfer rate up to a maximum of 20–23% for the ranges of K^* and Reynolds number studied.

NOMENCLATURE

b, thickness of the test cylinder wall [m];
c, constant in equation (4);
D, outside diameter of the test cylinder [m];
h, heat-transfer coefficient [$W/m^2 K$];
K, thermal conductivity [W/m K];
L, length [m];
n, constant in equation (4);
q, heat flux [W/m^2];
$\dot{q}$, heat generation per unit volume [W/m^3];
Q, heat [W];
R, outside radius [m];
t, temperature [K];
T, non-dimensional temperature, $(t_w - t_\infty)/(\bar{t}_w - t_\infty)$;
U, velocity [m/s];
K^*, dimensionless conduction parameter, $K_\infty R/K_t b$;
Nu, Nusselt number;
Re, Reynolds number;
Tu, turbulence intensity.

Greek symbols

ε, total emissivity;
η, dimensionless temperature, $(t_w - t_\infty)/(t_{st} - t_\infty)$;
θ, angle;
μ, viscosity [kg/sm];
ρ, density [kg/m^3];
σ, Stefan–Boltzmann constant; $5.67 \times 10^{-8}\ W/m^2 K^4$.

Superscript

$\bar{\ }$, mean.

Subscripts

McA, $\overline{Nu}$ reported in [11];
pres, $\overline{Nu}$ of this study;
st, front stagnation point;
t, test cylinder material;
w, at wall, local;
∞, free stream.

INTRODUCTION

RECENTLY, the local and average heat-transfer coefficients of a cylinder placed in cross-flow have been measured in the Reynolds number range up to 4×10^6 including surface roughness by Achenbach [1]. However, to have a complete similarity in the experimental results by the different investigators, all parameters governing this heat-transfer process should be the same. These parameters should include the stream turbulence intensity, the blockage ratio, the roughness ratio, etc., but seldom included is the circumferential heat conduction which is dependent on the thermal conductivity of the cylinder wall material, the wall thickness, the heat generation rate and the cylinder diameter.

Ideally, the effects of various parameters (except the last one mentioned above) on the heat-transfer distribution can only be studied if the heat-transfer data are obtained from a completely isothermal cylinder surface but this is not possible with the most of equipments used by many investigators. Therefore, Giedt [2] studied the effect of wide variation of the temperature distribution and temperature gradient around the cylinder circumference by using a thin nichrome ribbon wound helically around a cylinder which was electrically heated. The variation was introduced by varying the power input to the ribbon. Giedt concluded from his observation that the effect of the temperature variation along the ribbon on the local heat-transfer coefficients was practically negligible. However, he cautioned that this conclusion was far from established by his limited amount of data and he felt further investigation should be conducted.

On the other hand, marked differences in the distribution of the heat-transfer coefficient at inner and outer surfaces of the cylinder was observed by Thomson *et al.* [3]. They accounted this difference to the circum-

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ferential heat conduction and reported that as the tube thickness decreases, the disparity between outside and inside heat transfer rates tends to disappear.

Although the flow situations are different, it has been shown that circumferential wall conduction has a significant effect on both the local and average heat-transfer coefficients in a turbulent flow through eccentric annular ducts [4, 5].

An analytical study on the effect of wall thickness, thermal conductivity and method of heat input on the heat-transfer performance of some ribbed surfaces has also been reported by Barnett [6]. His conclusions were that the surface temperature varies significantly over a ribbed surface with the thickness and thermal conductivity of the surface material, but two methods of heat input (by heat generation within the wall and by heat conduction to the inner surface of the wall) do not give significantly different results.

The possibility that the different experimental set-up and techniques might account for some of the apparent variation in reported local and average heat-transfer coefficients of the cylinder placed in a cross-flow led to the experimental study presented in the present paper.

An analysis is possible for the local heat-transfer coefficient in both the laminar and turbulent boundary-layer regions of the tube surface using well established velocity and temperature profiles across the boundary layer for a free stream turbulence intensity. However, in the region of the separation, this is not now feasible. Therefore, an empirical approach to the problem seemed to be proper.

In the present study, a non-dimensional parameter $K^* = K_\infty R/K_t b$ which can be derived from the governing energy differential equation as shown in the Appendix, has been used to characterize the peripheral wall heat conduction.

An extensive review on the particular heat-transfer process can be found in the literature such as that of [7].

EXPERIMENTAL APPARATUS

The wind tunnel used for the experiment has a rectangular test-section of 260 × 150 mm and 800 mm in length. The air velocity could be varied up to 58 m/s by means of two controls, coarse and fine adjustable throttling gates located in the down-stream of the test-section. The free stream turbulence intensity could be varied by the numbers and sizes of the screens at the up-stream of the test-section but it was kept constant throughout the tests at about 4.0%.

A provision was included to measure the velocity distribution in two-dimensions at the test-section. The velocity measurement was done by a pitot-total tube with static pressure tappings on the wall of the test-section in conjunction with a micromanometer. The pressure measurement was also supplemented by a multitude inclined manometer.

Five test cylinders (electrical resistance heating) made of different materials and with different wall thickness were used (see Table 1). Outside diameters of the test cylinders were kept nearly closed to 25.4 mm to maintain a constant blockage ratio so that the aerodynamic part or flow pattern around the test-section would be the same for each test cylinder.

Table 1

No.	Material	O.D. (mm)	I.D. (mm)	K^*
1	Monel 400	25.8	18.8	1.64×10^{-3}
2	Stainless steel 304	25.3	18.7	7.10×10^{-3}
3	Inconel 600	25.8	21.1	9.45×10^{-3}
4	Stainless steel 304	25.4	22.9	19.2×10^{-3}
5	Stainless steel 304	25.4	23.7	28.9×10^{-3}

Sixteen K-junctions (chromel–alumel) were provided around the periphery of the test cylinders at the center which was 121 mm from either end of the electrical thermal connectors made of copper shrink fitted into the test cylinder. Two more K-junctions are embedded on the wall 76 mm away from the center of the test cylinder to check the uniformity of the temperature distribution. Therefore, the end effect of the temperature reading can be checked. Thirteen junctions measured the surface wall temperatures at the interval of 15° in the upper portion of the cylinder and three other junctions were located at the interval of 45° to check the symmetry in the temperature distribution on the upper and lower portions of the cylinder. All the thermocouple junctions on the test cylinders were made by using the "split hot junction method" [5].

The electrical power input to the test cylinder was measured with two copper leads embedded in the cylinder 127 mm apart in the central portion.

The test cylinders were placed horizontally in the rectangular test-section. Teflon supports were used for insulating electrically. All test parameters except K^* and Reynolds number were kept constant throughout the tests.

EXPERIMENTAL PROCEDURE AND DATA REDUCTION

All thermocouples used were calibrated *in situ*. After a preliminary run of the wind-tunnel fan, the flow control valves were adjusted to obtain the desired air velocity. When the air velocity became constant with respect to time, the power input to the test cylinder was regulated so that the difference between the average surface temperature and the free stream air temperature was between about 12–40°C. A duration of up to 60 min was normally required to reach the steady state condition.

The free stream air temperature and velocity, the power input into the test cylinder, and the wall surface temperature at 18 points around the circumference of the test cylinders were almost simultaneously recorded by a data acquisition system (Hewlett–Packard 5050 B in conjunction with HP 2901 and HP 2401 C). For temperature measurement, potentiometer strip chart recorders were also used with the data acquisition system.

The tests were carried out over the range of Reynolds numbers between 10^3 and 10^5 and the reproducibility of the experimental results was checked by repeating a number of test runs at the same condition.

Because of circumferential heat conduction, even with electrical resistance heating, the condition of constant heat flux is no longer applicable. Therefore, the local heat-transfer coefficient is deduced from equation (1) which was obtained from an energy balance made on the element of cylinder wall of Fig. A1, including radiation loss (see Appendix).

$$h = \frac{\dfrac{d^2 t_w}{d\theta^2} + \dfrac{\dot{q}R^2}{K_t}}{\dfrac{R^2}{K_t b}(t_w - t_\infty)} - \varepsilon\sigma(t_w^2 + t_\infty^2)(t_w + t_\infty). \qquad (1)$$

This is very similar to the method used by Giedt [2] for the calculation of local heat-transfer coefficient. A value of 0.2 for ε was used here as suggested by Giedt [2].

The values of $(d^2 t_w/d\theta^2)$ were determined graphically from the experimental curves of circumferential temperature distribution such as illustrated in Fig. 2. These h from equation (1) are used to calculate the local Nusselt numbers defined as

$$Nu = hD/K. \qquad (2)$$

The average Nusselt number, $\overline{Nu}$, was obtained by integration of the local Nusselt numbers, Nu, over the circumference of the cylinder. The average Reynolds number is calculated from the definition

$$Re = \frac{\rho D U_\infty}{\mu}. \qquad (3)$$

RESULTS AND DISCUSSION

The free stream velocity over the test-section, located in the central portion of the tunnel must be uniform. This has been verified by velocity measurement across the test-section at different values of Reynolds numbers

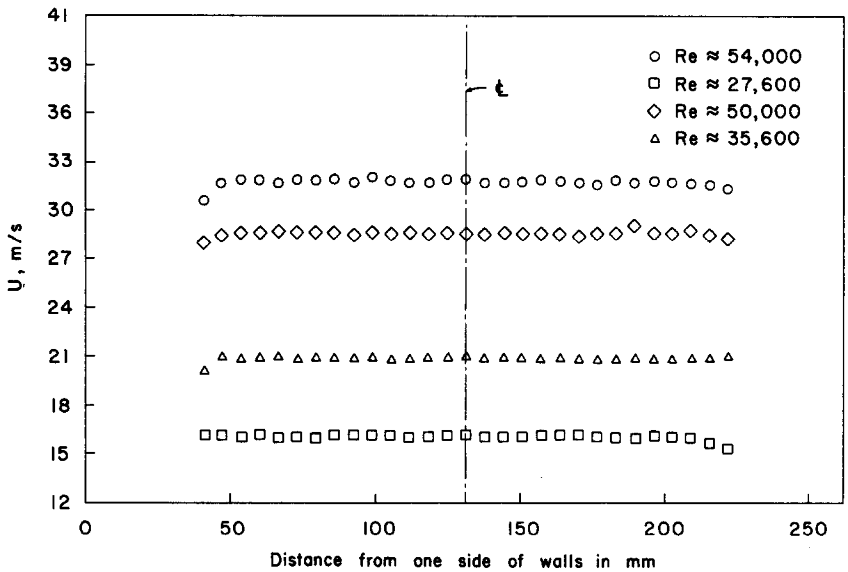

FIG. 1. Typical velocity measurement.

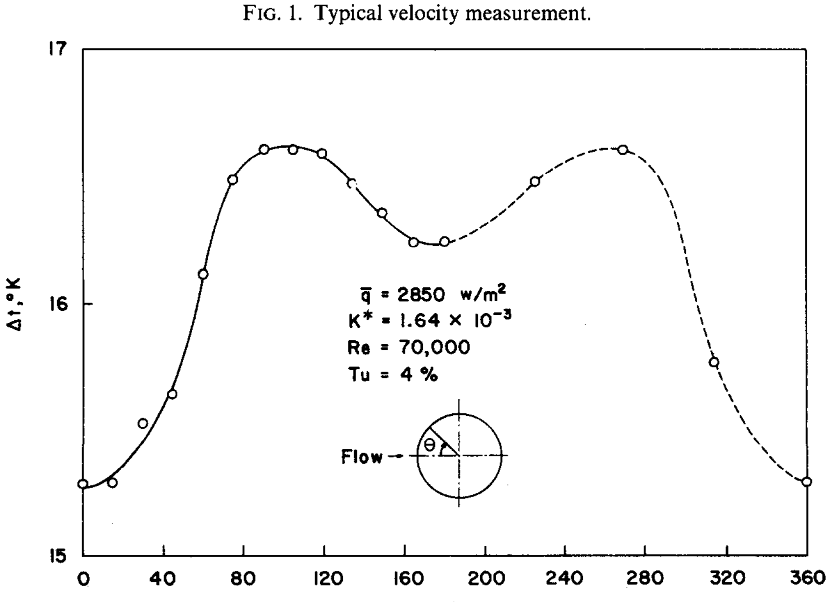

FIG. 2. A typical circumferential temperature distribution.

and some of the measurements are shown in Fig. 1 as examples. The vertical velocity measurement also showed similar pattern.

An example of the circumferential surface temperature of the test cylinder at 16 points is shown in Fig. 2. As can be seen from the figure, there is no significant difference between measurements at symmetric angles on the upper and lower portions of the test cylinder.

A representative local temperature distribution, in dimensionless form of T vs θ at constant Reynolds number, is shown in Fig. 3, which clearly exhibits the effect of parameter K^*. The measurement in the Reynolds number range between 10^3 and 10^5 shows that the minimum value of T occurs always at the forward stagnation point, $\theta = 0°$ but the maximum value of T in the range of angles between 70 and 130° and they are always greatly influenced by the value of K^* as illustrated in Fig. 3. The high value of K^* increases the variation of dimensionless temperature, T, around the cylinder surface (large values of K^* correspond to poor thermal conductors). It was also observed that the increased rate of peripheral conduction in the cylinder wall (corresponding to lower values of K^*) augmented the mean temperature level of the cylinder surface, slightly.

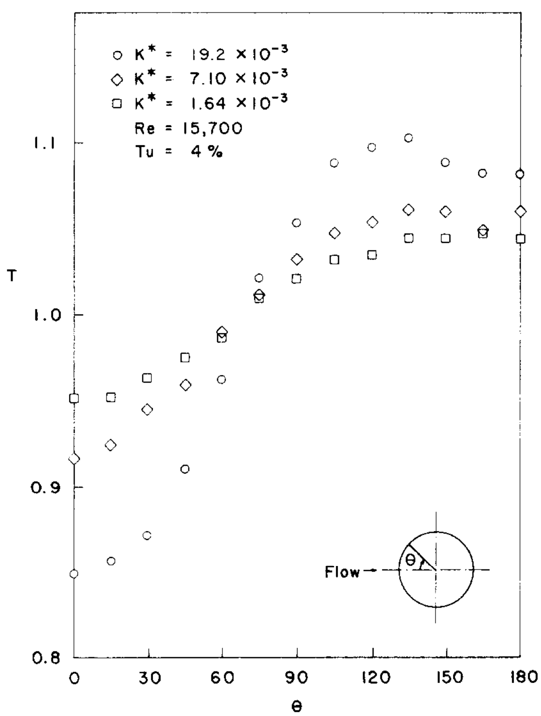

FIG. 3. Non-dimensional circumferential wall temperature distribution.

For the conditions of constant heat flux generation, the maximum local wall temperature at a given flow condition is greatly affected by the value of K^* and this fact can be of practical importance. As the value of K^* increases, the maximum local wall temperature also increase and there is a possibility of forming a hot spot which may be undesirable.

To show the effect of K^* qualitatively, non-dimensional temperature distribution was obtained by solving the differential equation obtained from energy balance made on the element of the cylinder wall (see Appendix). Solutions were obtained by employing CSMP (Continuous System Modeling Program) technique. The numerical results with prescribed fictional heat flux distributions also confirmed the general pattern of change in T vs θ with respect to the values of K^* but these results are not reported quantitatively (as meaningful conclusions could only be obtained by using the heat flux distribution close to the reality in the differential equation which is not possible at the moment).

The typical local and average heat-transfer coefficients in Nusselt number are illustrated in Fig. 4 for

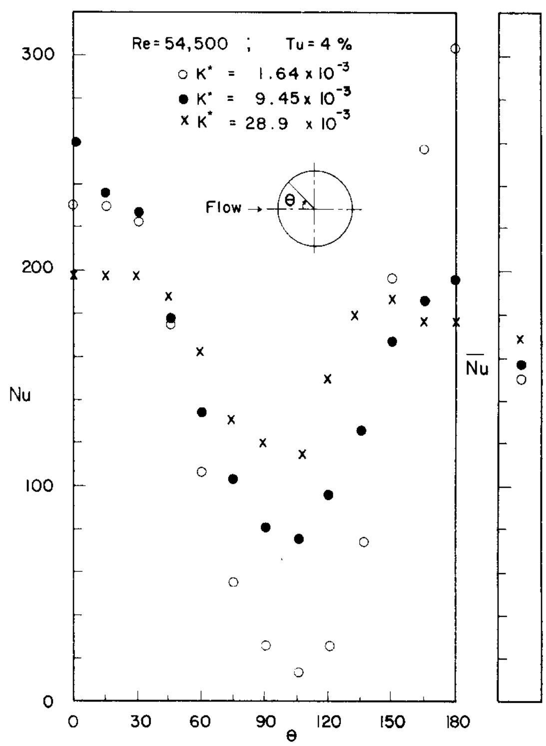

FIG. 4. Local and average heat-transfer coefficients.

a Reynolds number of 54500 for three values of K^* ranging from 1.64×10^{-3} to 28.9×10^{-3}.

Contrary to the trend seen in Fig. 3, the low value of K^* increases the variation of local Nusselt number around the test cylinder surface. This trend can also be seen in Fig. 5. However, the effect of parameter K^* on the overall average Nusselt number seems to be moderate; the difference in average Nusselt numbers for the case in Fig. 4 being about 17%, which is comparable to the results obtained in a symmetric flow with no boundary layer separation [4, 5].

The present local values are also compared in Fig. 5 with some of the previous investigators. Most works in this particular field of study do not specify the physical information that is needed to calculate the parameter K^*. Therefore, the comparison of the present work with those of other investigators is merely qualitative.

Schmidt and Wenner [8] measured the local heat-transfer coefficient around the cylinder with an iso-

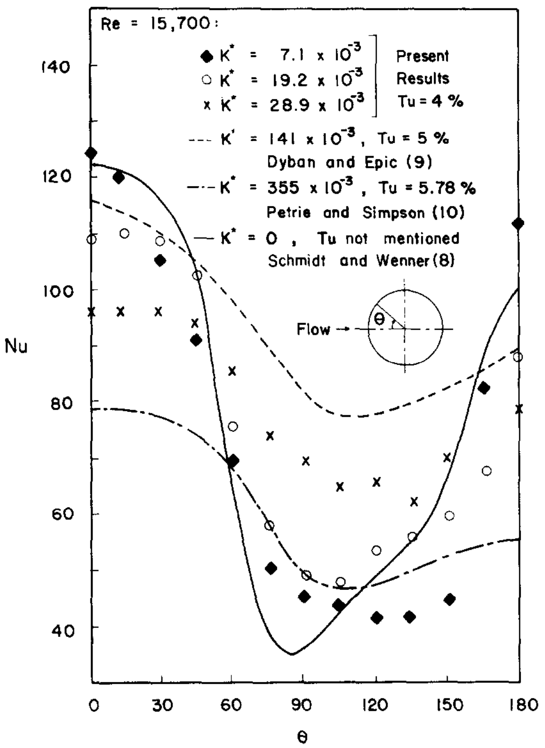

FIG. 5. Local heat-transfer coefficients.

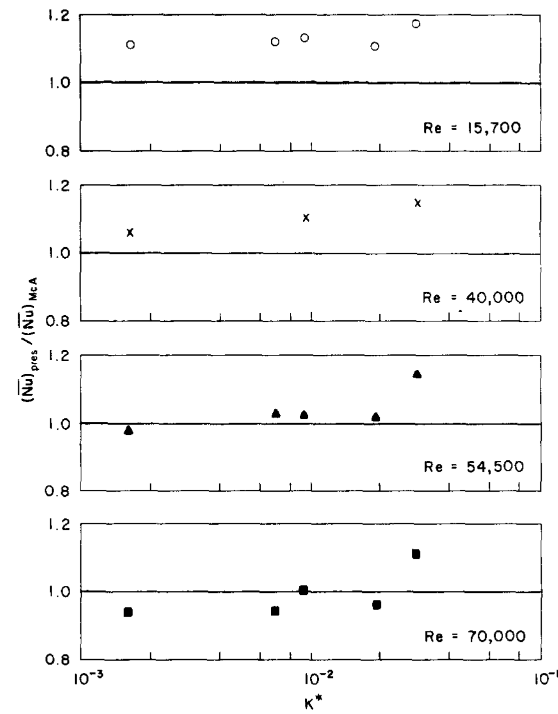

FIG. 6. Effect of K^* on average Nu.

thermal surface and we put the value of K^* as 0. The value of turbulence intensity was not reported.

Both Dyhan and Epick [9], and Petrie and Simpson [10], obtained their data from constant heat flux condition and from their papers, their values of parameter K^* were best estimated at 141×10^{-3} and 355×10^{-3}, respectively. From Fig. 5 and also from similar comparisons made, it has been observed that when the test physical conditions are similar, the general shape of the distribution of local Nusselt numbers and the effect of parameter K^* on local Nu were also similar to those of the present work.

To see the effect of parameter K^* together with Reynolds number, the average Nusselt number obtained from the present study is non-dimensionalized by dividing with the generally accepted formula [11] as given by

$$\overline{Nu}_{\mathrm{McA}} = CRe^{n} \tag{4}$$

where C and n are constants, a function of Reynolds number.

These values are compared against the parameter K^* as shown in Fig. 6. The effect of the parameter K^* seems to be more pronounced with high Reynolds number. As much as 20% increase in average Nusselt number can be observed in Fig. 6 as the parameter K^* was increased from 1.64×10^{-3} to 28.9×10^{-3} at $Re \simeq 70\,000$. This is quite well in accord with the recent findings of Boulos and Pai [12] who concluded that the overall heat-transfer rate (from a circular cylinder placed normal to a turbulent air stream) under constant heat flux conditions is about 10–20% higher than that under constant temperature conditions (corresponding $K^* \simeq 0$).

CONCLUSIONS

The following conclusions may be drawn from the present study:

1. The large values of the parameter K^* especially at high Reynolds number are associated with large variation of wall temperature. Therefore, for the constant heat flux generation, with high values of K^*, there may be undesirable hot spots which could be beyond the allowable wall design temperature.

2. The added asymmetry of the thermal boundary condition due to the value of parameter K^* affects the average heat transfer rate up to a maximum of 20–23% for the ranges of the parameter K^* and Reynolds number studied.

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APPENDIX

For unit length of test tube idealized in Fig. A1, an energy balance is made as

$$Q_1 + Q_i = Q_2 + Q. \tag{A1}$$

Here, the radiation loss is assumed to be negligible. In differential form, equation (A1) becomes

$$\frac{d^2 t_w}{d\theta^2} - \frac{qR^2}{K_t b} + \frac{\dot{q}R^2}{K_t} = 0. \tag{A2}$$

With non-dimensional temperature η and $\bar{\eta}$ defined as

$$\eta = \frac{t_w - t_\infty}{t_{st} - t_\infty} \quad \text{and} \quad \bar{\eta} = \frac{\bar{t}_w - t_\infty}{t_{st} - t_\infty}$$

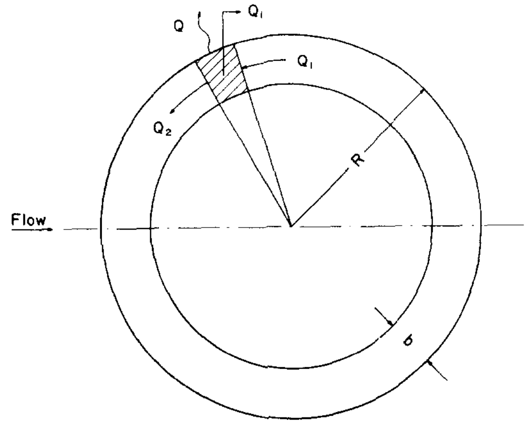

FIG. A1. Idealized model.

equation (A2) can be rewritten as

$$\frac{d^2\eta}{d\theta^2} - \frac{q}{(t_{st} - t_\infty)} \frac{R^2}{K_t b} + \frac{\dot{q}R^2}{(t_{st} - t_\infty)K_t} = 0. \tag{A3}$$

Now from energy balance, we obtain

$$\dot{q} = \frac{\bar{q}}{b}$$

and with the definition of $\bar{h}$, given as

$$\bar{q} = \bar{h}(\bar{t}_w - t_\infty).$$

Equation (A3) reduces to

$$\frac{d^2\eta}{d\theta^2} - \frac{K^*}{2} \overline{Nu}\bar{\eta}\left[\frac{q}{\bar{q}} - 1\right] = 0. \tag{A4}$$

EFFET DE LA CONDUCTION PARIETALE PERIPHERIQUE SUR LE TRANSFERT DE CHALEUR AUTOUR D'UN CYLINDRE EN ATTAQUE TRANSVERSALE

Résumé—La paroi du cylindre placé dans un écoulement transversal étant le siège d'une source de chaleur uniforme, la chaleur est transmise suivant la périphérie par conduction du fait de la nature asymétrique de l'écoulement autour du cylindre. La circulation périphérique de la chaleur affecte la distribution de température pariétale dans une telle mesure que, dans certains cas, des résultats notablement différents peuvent être obtenus pour des surfaces géométriquement semblables.

Dans la présente étude, un paramètre adimensionnel $K^* = K_\infty R/K_t b$ a été introduit pour caractériser la conduction thermique pariétale périphérique. Dans l'expérimentation, cinq cylindres d'essai ont été utilisés présentant des valeurs différentes de K^* (allant de 0,00164 à 0,0289), pour des nombres de Reynolds, basés sur le diamètre du tube, variant depuis 1000 jusqu'à 100 000; l'intensité de turbulence reste fixée à 4 pour cent dans l'écoulement libre.

Les valeurs élevées de K^*, en particulier aux grands nombres de Reynolds, sont associées aux variations importantes de température de paroi. La dissymétrie additionnelle des conditions aux limites thermiques imposées affecte le taux de transfert de chaleur moyen dans une proportion atteignant 20–23 pour cent, dans les domaines de valeurs de K^* et des nombres de Reynolds étudiés.

DER EINFLUSS DER PERIPHEREN WÄRMELEITUNG IN DER WAND EINES QUERANGESTRÖMTEN ZYLINDERS AUF DEN WÄRMEÜBERGANG

Zusammenfassung—Bei einheitlicher Wärmeerzeugung in der Wand eines querangeströmten Zylinders wird infolge der asymmetrischen Natur des Strömungsfeldes um den Zylinder Wärme in Umfangsrichtung geleitet. Der periphere Wärmestrom beeinflußt die Temperaturverteilung in der Wand so erheblich, daß in einigen Fällen für ähnliche Geometrien stark unterschiedliche Ergebnisse erhalten werden.

In der vorliegenden Arbeit wird die periphere Wärmeleitung in der Wand durch einen dimensionslosen Parameter $K^* = K_\infty R/K_t b$ erfaßt. Bei der experimentellen Untersuchung wurden fünf Versuchszylinder mit unterschiedlichen Werten von K^* (0,00164 bis 0,0289) verwendet; die mit dem Zylinderdurchmesser gebildete Reynolds-Zahl variierte zwischen 1000 und 100 000, der Turbulenzgrad in der freien Strömung betrug konstant 4,0%.

Bei großen Werten K^* treten starke Unterschiede in der Wandtemperatur auf, insbesondere bei hohen Reynolds-Zahlen. Zusammen mit der zusätzlichen Asymmetrie der thermischen Grenzschichtbedingungen wurde im untersuchten Bereich der Reynolds-Zahlen und K^* der mittlere Wärmestrom bis zu maximal 20 bis 23% verändert.

ВЛИЯНИЕ ТЕПЛОПРОВОДНОСТИ СТЕНКИ ПО ОКРУЖНОСТИ НА ПЕРЕНОС ТЕПЛА ОТ ЦИЛИНДРА ПРИ ПОПЕРЕЧНОМ ОБТЕКАНИИ

Аннотация — При равномерном выделении тепла в стенке цилиндра в поперечном потоке кондуктивный перенос тепла происходит по окружности благодаря асимметричному характеру обтекания цилиндра по периметру. Тепловой поток по окружности влияет на распределение температуры стенки в такой степени, что в отдельных случаях для геометрически одинаковых поверхностей могут быть получены сильно отличающиеся результаты. В данном исследовании для описания теплопроводности стенки по окружности использовался безразмерный параметр $K^* = K_\alpha R/K_t b$. Во время экспериментального исследования использовались пять цилиндров с различными величинами K^* (от 0,00164 до 0,0289) в диапазоне чисел Рейнольдса, отнесенных к диаметру трубы, от 1000 до 100 000 при постоянной интенсивности турбулентности свободного потока 4%. Найдено, что большие значения параметра K^*, особенно при больших числах Рейнольдса, связаны с большими изменениями температуры стенки. Дополнительная асимметрия теплового граничного условия ведет к увеличению скорости переноса тепла до 20–23% в диапазоне исследованных чисел Рейнольдса и параметра K^*.